MATH-819 Analysis of Fractional Differential Equations

Credit Hours: 3-0
Prerequisite: None

Objectives and Goals: The aim of the course is to motivate students to study different topicsof the theory of fractional calculus and fractional differential equations.

Core Contents: BVPs for Nonlinear Second-Order ODEs, Existence and Uniqueness Theorems, Riemann–Liouville Differential and Integral Operators, Riemann–Liouville Differential and Integral Operators, Caputo's Approach, Mittag-Leffler Functions, Existence and Uniqueness Results for Riemann–Liouville and Caputo Fractional Differential Equations

Detailed Course Contents: BVPs for Nonlinear Second-Order ODEs: Contraction Mapping Theorem, Application of the Contraction Mapping Theorem to a Forced Equation Application of Contraction Mapping Theorem to BVPs, Lower and Upper Solutions,Nagumo Condition.
Existence and Uniqueness Theorems: Basic Results, Lipschitz Condition and Picard-Lindelof Theorem, Equicontinuity and the Ascoli-Arzela Theorem, Cauchy-Peano Theorem, Extendability of Solutions, Basic Convergence Theorem, Continuity of Solutions with Respect to ICs, Kneser's Theorem, Differentiating Solutions with Respect to ICs, Maximum and Minimum Solutions.
Introduction to Fractional Calculus: Motivation, The Basic Idea, An Example Application of Fractional Calculus. Riemann–Liouville Differential and Integral Operators: Riemann–Liouville Integrals, Riemann–Liouville Derivatives, Relations Between Riemann–Liouville Integrals and Derivatives, Grunwald–Letnikov Operators. Caputo's Approach: Definition and Basic Properties, Nonclassical Representations of Caputo Operators. Mittag-Leffler Functions: Definition and Basic Properties.
Existence and Uniqueness Results for Riemann–Liouville Fractional Differential Equations. Single- Term Caputo Fractional Differential Equations: Existence of Solutions, Uniqueness of Solutions, Influence of Perturbed Data, Smoothness of the Solutions, Boundary Value Problems. Advanced Results for Special Cases: Initial Value Problems for Linear Equations,Boundary Value Problems for Linear Equations, Stability of Fractional DifferentialEquations.

Course Outcomes: Students are expected to understand:
- Existence theory for second order ordinary differential equations
- Properties of fractional operators
- Existence theory of fractional differential equations

Text Books:

1. Walter G. Kelley, Allan C. Peterson, Theory of Differential Equations, Second Edition,Springer, (2010) (Referred as KP).
2. Kai Diethelm, The Analysis of Fractional Differential Equations, Springer, 2010)(Referred asKD).

Reference Books:

1. Podlubny, Fractional Differential Equations. Academic Press, San Diego (1999).
2. R. Hilfer, Applications of Fractional Calculus in Physics, World Scientific Publishing (2000).
3. A.A. Kilbas, H.M. Srivastava, J.J. Trujillo, Theory and applications of fractional differential equations, vol 204. North-Holland mathematics studies. Elsevier, Amsterdam (2006).

ASSESSMENT SYSTEM

Nature of assessment	Frequency	Weightage (%age)
Quizzes	Minimum 3	10-15
Assignments	-	5-10
Midterm	1	25-35
End Semester Examination	1	40-50
Project(s)	-	10-20

Weekly Breakdown

Week	***Section***	***Topics***
1	7.1,7.2 (KP)	BVPs for Nonlinear Second-Order ODEs: Contraction Mapping Theorem, Application of the Contraction Mapping Theorem to a Forced Equation.
2	7.3-7.5	Application of Contraction Mapping Theorem to BVPs, Lower and Upper Solutions, Nagumo Condition.
3	8.1-8.3	Existence and Uniqueness Theorems: Basic Results, Lipschitz Condition and Picard-Lindelof Theorem, Equicontinuity and the Ascoli-Arzela Theorem.
4	8.4-8.6	Cauchy-Peano Theorem, Extendability of Solutions, Basic Convergence Theorem
5	8.7-8.10	Continuity of Solutions with Respect to ICs, Kneser's Theorem, Differentiating Solutions with Respect to ICs, Maximum and Minimum Solutions.
6	1.1-1.3 (KD)	Introduction to Fractional Calculus: Motivation, The Basic Idea, An Example Application of Fractional Calculus.
7	2.1, 2.2	Riemann–Liouville Differential and Integral Operators: Riemann–Liouville Integrals, Riemann–Liouville Derivatives.
8	2.3, 2.4	Relations Between Riemann–Liouville Integrals and Derivatives, Grünwald–Letnikov Operators.
9	**Mid Semester Exam**	
10	3.1, 3.2	Caputo's Approach: Definition and Basic Properties, Nonclassical Representations of Caputo Operators.

11	4	Mittag-Leffler Functions: Definition and Basic Properties.
12	5	Existence and Uniqueness Results for Riemann–Liouville Fractional Differential Equations.
13	6.1, 6.2	Single-Term Caputo Fractional Differential Equations: Basic Theory and Fundamental Results: Existence of Solutions, Uniqueness of Solutions.
14	6.3, 6.4	Influence of Perturbed Data, Smoothness of the Solutions
15	6.5	Boundary Value Problems.
16	7.1-7.3	Advanced Results for Special Cases: Initial Value Problems for Linear Equations, Boundary Value Problems for Linear Equations, Stability of Fractional Differential Equations.
17		Review
18	**End Semester Exam**	